

$$\beta_{\alpha} = \frac{\partial \ln \mathcal{Z}}{\partial \ln \beta} = \frac{\partial}{\partial \ln \beta} \left(\ln \int \mathcal{D}\phi \exp(-\beta \mathcal{H}[\phi]) \right) = - \frac{\partial}{\partial \ln \beta} \left(\frac{1}{\mathcal{Z}} \int \mathcal{D}\phi \mathcal{H}[\phi] \exp(-\beta \mathcal{H}[\phi]) \right) = - \frac{\partial}{\partial \ln \beta} \langle \mathcal{H} \rangle = \langle \mathcal{E} \rangle$$
$$\beta_{\alpha} = \frac{\partial \ln \mathcal{Z}}{\partial \ln \beta} = \frac{\partial}{\partial \ln \beta} \left(\ln \int \mathcal{D}\phi \exp(-\beta \mathcal{H}[\phi]) \right) = - \frac{\partial}{\partial \ln \beta} \left(\frac{1}{\mathcal{Z}} \int \mathcal{D}\phi \mathcal{H}[\phi] \exp(-\beta \mathcal{H}[\phi]) \right) = - \frac{\partial}{\partial \ln \beta} \langle \mathcal{H} \rangle = \langle \mathcal{E} \rangle$$